1.1 Crab Waist Collision Scheme

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1.1.1 We do we need a new collision scheme?

The luminosity L is a figure of merit of a collider. The rate of useful physics events produced in collision is proportional to this parameter:

$$\dot{N} = \sigma_p L \qquad (1)$$

where σ_p is the cross section of the physics process. Clearly that in order to increase the rate of rare events with a small cross section the highest possible luminosity is required. For standard collision schemes with head-on collisions the luminosity is expressed by the following formula:

$$L = f_0 \frac{N_b}{4\pi} \frac{N^2}{\sigma_x \sigma_y} \qquad (2)$$

N_b is the number of bunches, f_0 the revolution frequency. Here for simplicity we assume that the colliding bunches have equal intensities N and equal horizontal σ_x and vertical beam sizes σ_y at the collision point:

$$\sigma_x = \sqrt{\varepsilon_x \beta_x^*}; \qquad \sigma_y = \sqrt{\varepsilon_y \beta_y^*} \qquad (3)$$

where $\varepsilon_{x,y}$ are the horizontal and the vertical beam emittances and $\beta^*_{x,y}$ are the respective betatron optics functions at the collision point.

As it is seen from the luminosity definition (2), for a given collider circumference and a fixed number of colliding bunches the only way to increase the luminosity is to increase the bunch intensities and/or to reduce the transverse beam sizes. However, in the standard collision scheme the luminosity increase is limited by both the beam dynamics and collider optics requirements.

First of all, the colliding bunches experience strong electromagnetic interaction in collision. The electron and positron bunches focuses each other in a very nonlinear manner. The strength of such a nonlinear beam-beam lens is characterized by the so called "beam-beam tune shift parameter" $\xi_{x,y}$ that is proportional to the shift of frequencies of the transverse (betatron) oscillations due to the beam-beam interaction. For flat bunches of lepton colliders we have:

$$\xi_{x,y} = \frac{r_e N \beta_{x,y}}{2\pi\gamma\sigma_{x,y}\left(\sigma_x + \sigma_y\right)} \qquad (4)$$

Comparing eq.(2) and (4) we see that by trying to increase the luminosity at the expense of higher N and lower $\sigma_{x,y}$ we also increase the strength of the nonlinear beam-beam interaction thus forcing nonlinear beam-beam resonances, increasing spread of the betatron frequencies, creating dynamical chaos etc. This leads to collider performance degradation: beams blowup, lifetime reduction, appearance of beam-beam instabilities and other harmful effects. The problem can be solved if we find a way to linearize or weaken the strong nonlinear beam-beam interaction.

Yet another problem limiting the luminosity comes from the “hour-glass” effect. The luminosity formula (2) is valid in the limit of an infinitely short bunch. However, in a collider bunches have a finite length σ_z and it is rather difficult to shorten the bunch in high intensity colliders without incurring into beam instabilities. Besides, too short bunches tend to produce excessive power losses due to parasitic interaction with the vacuum chamber beam coupling impedance and/or due to eventual coherent synchrotron radiation. So, one has to take into account the luminosity reduction due to the dependence of the transverse beam size along the bunch length (“hour-glass effect”), as shown in Fig. 1. As we can see, the minimum transverse beam size is reached only at the collision focal point (“beam waist”). Then, the transverse size starts rapidly growing towards the bunch tails, proportionally to the square root of the beta function $\beta_{x,y}$ (see eq. (3)), so that the effective luminosity becomes lower. The luminosity reduction due to this effect gets substantial when the beta function becomes comparable with the bunch length σ_z. For lower beta, β_y < σ_y, the luminosity reduction is even more pronounced. In addition, in such conditions new harmful synchro-betatron resonances are excited by the beam-beam interaction.

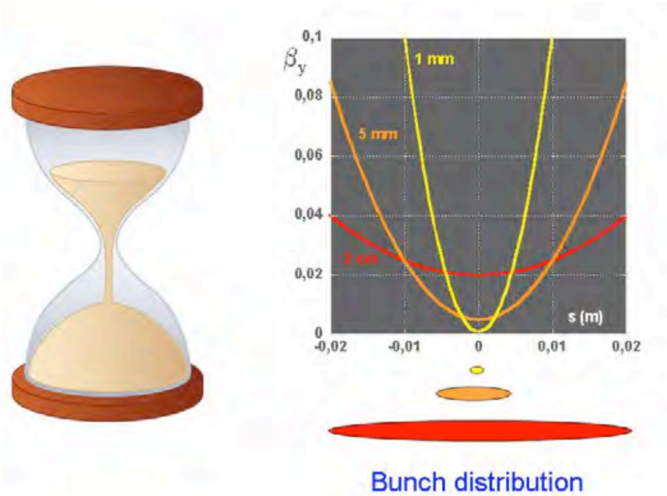
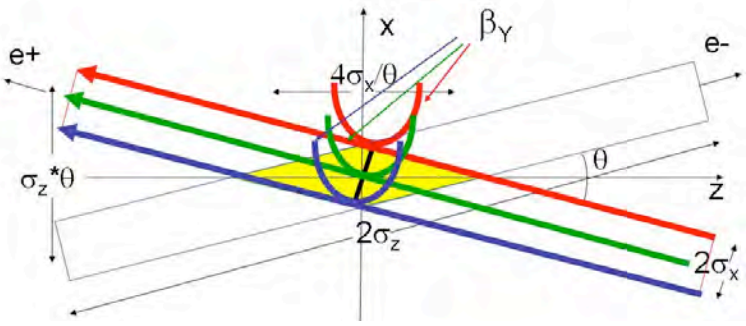


Fig. 1 The hour-glass effect

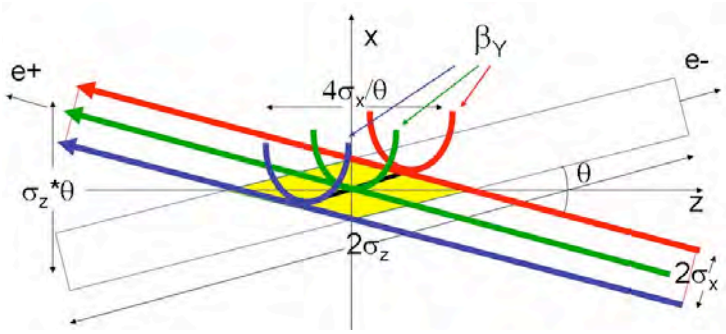
The proposed at Frascati [1,2] and successfully tested at DAΦNE [3] the Crab Waist collision scheme helps both to solve the problem of the strong nonlinear interaction of colliding beams and to alleviate the reduction of luminosity due to the “hour-glass” effect. This allows pushing the achievable luminosities far beyond the state-of-art values. At present this scheme is considered to be most attractive for the next generation lepton factories [4, 5].

1.1.2 Crab waist collision scheme in 3 steps

The CW scheme can substantially increase collider luminosity since it combines several potentially advantageous ideas. Let us consider two flat bunches ($\sigma_y << \sigma_x$) colliding under a horizontal crossing angle θ (as shown in Fig. 2a). Then, the CW principle can be explained in the three basic steps.



a) Crab sextupoles OFF.



b) Crab sextupoles ON.

Figure 2: Crab Waist collision scheme.

The **first one** requires a large Piwinski angle $\Phi = (\sigma_z/\sigma_x)tg(\theta/2) >> 1$. This characteristic parameter enters in the expressions for the luminosity, beam-beam tunes shifts and strengths of synchro-betatron resonances excited by beam-beam interaction. In particular, the luminosity and the tunes shifts in case of collisions with the horizontal angle can be written by simply substituting the horizontal beam size σ_x^* by the effective transverse size $\sigma_x^*(1+\Phi^2)^{1/2}$ [6]:

$$L = \frac{N_b f_0}{4\pi\sigma_x\sigma_y}\left[\frac{N^2}{\sqrt{1+\Phi^2}}\right]; \quad \xi_y = \frac{r_e\beta_y}{2\pi\gamma\sigma_x\sigma y}\left[\frac{N}{\sqrt{1+\Phi^2}}\right]; \quad \xi_x = \frac{r_e\beta_x}{2\pi\gamma\sigma_x^2}\left[\frac{N}{1+\Phi^2}\right] \quad (5)$$

We see that for large Piwinski angle, Φ>>1, the luminosity and the tune shifts scale as:

$$L \propto \frac{N\xi_y}{\beta_y^*}; \quad \xi_y \propto \frac{N\sqrt{\beta_y / \varepsilon_y}}{\sigma_z\theta}; \quad \xi_x \propto \frac{N}{(\sigma_z\theta)^2} \qquad (6)$$

In such a case, it is possible to increase *N* proportionally to $\sigma_z\theta$, keeping the vertical tune shift ξ_y constant, while the horizontal tune shift will even drop proportionally to $1/\sigma_z\theta$. The resulting luminosity will grow proportionally to $\sigma_z\theta$.

In the CW scheme the Piwinski angle is increased by decreasing the horizontal beam size and increasing the crossing angle. In this way we can gain in luminosity and the horizontal tune shift decreases, as discussed above. But the most important feature is that the overlap region of the colliding bunches is reduced, since it is proportional to σ_x/θ (see Fig. 2). Then, as the **second step**, the vertical beta function β_y can be made comparable to the overlap area size (i.e. much smaller than the bunch length):

$$\beta_y^* \approx \frac{2\sigma_x}{\theta} \cong \frac{\sigma_z}{\Phi} << \sigma_z \qquad (7)$$

This is a very strong and beneficial factor since for the lower beta at the same time function the luminosity grows and the beam-beam nonlinear force becomes weaker. So, reducing β_y^* at the interaction point gives us several advantages (see the scaling (6)):

- Luminosity increase with the same bunch current.
- Possibility of the bunch current increase (if it is limited by ξ_y), thus farther increasing the luminosity.
- Suppression of the vertical synchrobetatron resonances [7].

Besides, there are additional advantages in such a collision scheme: there is no need in decreasing the bunch length to increase the luminosity as required in standard collision schemes. This will certainly helps solving the problems of HOM heating, coherent synchrotron radiation of short bunches, excessive power consumption, etc.

As the **third step**, further luminosity boost is provided by the crab waist transformation that performs a rotation of the beam waist position with respect to the particles motion. As it is seen in Fig. 2b, the beta function waist of one beam is now oriented along the central trajectory of the other one. Namely, due to such an unusual waist rotation the scheme is called "crab waist" using the analogy with the "crab crossing" scheme exploiting special crab cavities [8]. Differently from the crab crossing scheme where bunches are tilted by the crab cavities with respect to the beam longitudinal axis, CW rotates the optics function β_y.

In practice the CW vertical beta function rotation is provided by sextupole magnets placed on both sides of the IP in phase with the IP in the horizontal plane and at $\pi/2$ in the vertical one (as shown in Fig. 3).

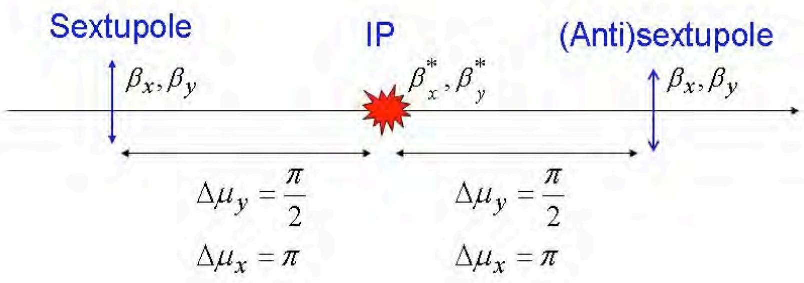


Figure 3. Crab sextupole locations.

The crab sextupole strength should satisfy the following condition depending on the crossing angle and the beta functions at the IP and the sextupole locations:

$$K = \frac{1}{\theta}\frac{1}{\beta_y^*\beta_y}\sqrt{\frac{\beta_x^*}{\beta_x}}$$

The crab waist transformation gives a small geometric luminosity gain due to the vertical beta function redistribution along the overlap area. It is estimated to be of the order of several percent. However, the dominating effect comes from the suppression of betatron (and synchrobetatron) resonances arising (in collisions without CW) due to the vertical motion modulation by the horizontal betatron oscillations [9].
Figure 5 shows luminosity tune scans, i.e. the luminosity as a function of the horizontal (ν_x) and vertical (ν_y) normalized betatron frequencies, for the two typical cases: a) standard scheme of collisions with the low Piwinski angle $\Phi < 1$ and vertical beta function β_y comparable with the bunch length σ_z (as in KEKB and DAΦNE before upgrade) and b) crab waist collisions with large Piwinski angle $\Phi >> 1$ and β_y comparable to the small overlap area σ_x/θ (as in SuperB, SuperC-Tau).

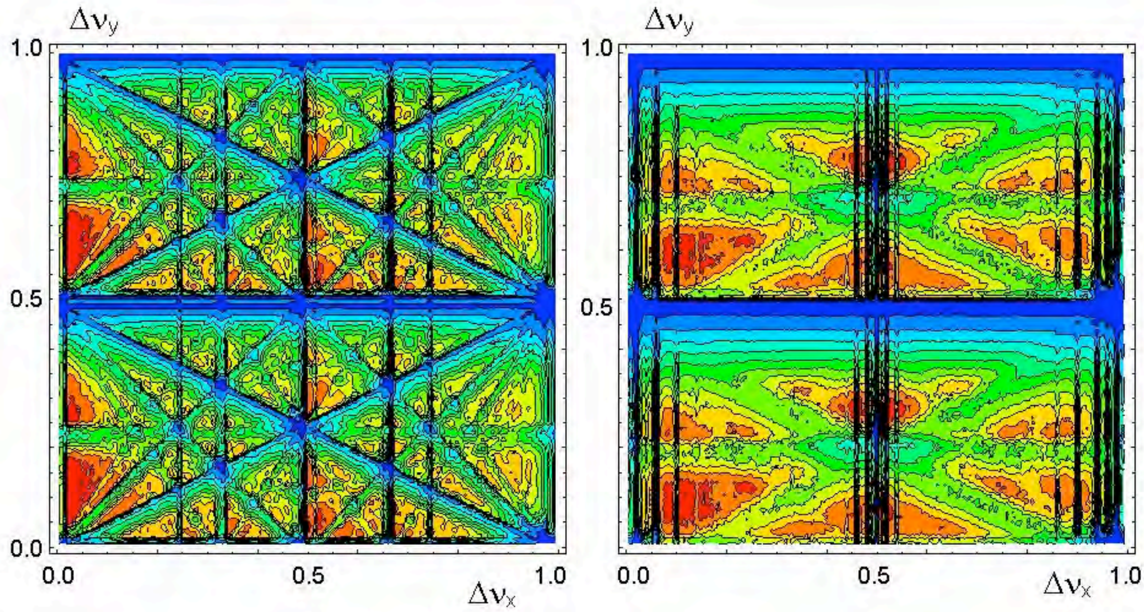


Figure 5. Luminosity scans for:
conventional collision scheme (left); crab waist collision scheme (right).

As it is seen in Fig.5, in crab waist collision X-Y coupling beam-beam resonances are successfully damped. Namely these resonances are considered to be most dangerous for flat colliding beams of the electron-positron factories leading to the strong beam blowup and beam lifetime degradation.

The crab waist collision scheme has been successfully tested at the electron-positron collider Φ-factory DAΦNE providing luminosity increase by a factor of 3, in a good agreement with numerical simulations [3]. Fig. 6 demonstrates the effect of the crab sextupoles on the beam-beam blow up and the distribution tails in DAΦNE.

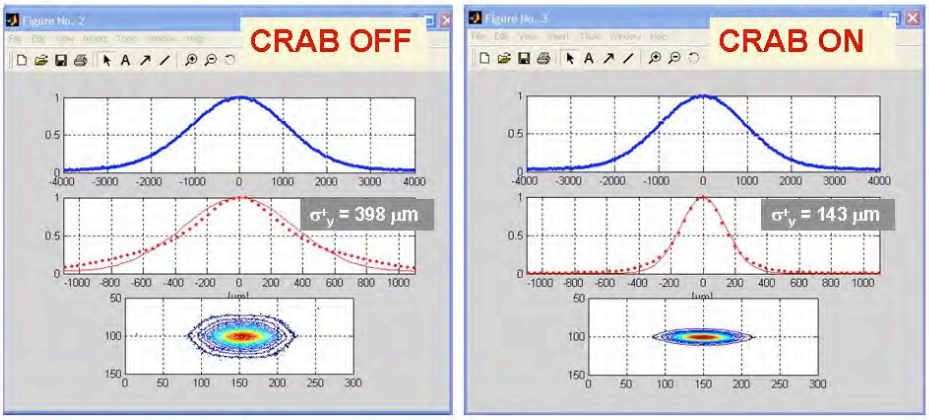


Figure 6. Transverse beam profiles with crab on and off

1.1.3 References

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